LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – NOVEMBER 2011

MT 3501/MT 3500 - ALGEBRA, CALCULUS AND VECTOR ANALYSIS

Date : 01-11-2011 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

PART-A

ANSWER ALL QUESTIONS **(10×2=20)**

1. Integrate $\int_0^1 \int_0^{1-x} ydydx$.
2. Prove that $\beta(m,n) = \beta(n,m)$.
3. Find the PDE by eliminating f from $z = f\left(x^2 - y^2\right)$.
4. Solve $p + q = \sin x + \sin y$.
5. Find the unit normal vector to the surface $x^2 + xy + z^2 = 4$ at the point $(1,-1,2)$.
6. State Green's theorem.
7. Find $L\left(e^{2t}t^2\right)$.
8. Evaluate $\int_0^\infty e^{-2t} \sin 3tdt$.
9. Define Euler's function.
10. State Fermat's theorem.

PART-B

ANSWER ANY FIVE QUESTIONS **(5×8=40)**

11. Change the order of integration and hence evaluate $\int_1^3 \int_0^{\frac{6}{x}} x^2 dydx$.

12. Using Gamma functions evaluate $\int_0^\infty e^{-x^2} dx$.

13. Solve given that $\frac{\partial^2 z}{\partial x^2} = a^2 z$ given that when x=0, $\frac{\partial z}{\partial x} = a \sin y, \frac{\partial z}{\partial y} = 0$.

14. Solve: $\left(x^2 - yz\right)p + \left(y^2 - zx\right)q = z^2 - xy$.

15. Find (i) $L(\sin 2t \sin 3t)$ (ii) $L(t \cos 3t)$.

16. Find $L^{-1}\left(\frac{5s+3}{(s-1)\left(s^2+2s+5\right)}\right)$.

17. Prove that $\vec{F} = \left(y^2 \cos x + z^3\right)\vec{i} + \left(2y \sin x - 4\right)\vec{j} + 3xz^2\vec{k}$ is irrotational and find its scalar potential.

18. Show that 18! +1 is divisible by 437.

PART-C

ANSWER ANY TWO QUESTIONS **(2 X 20=40)**

19. a) Evaluate $\iint_A xydxdy$, where A is the region bounded by x=2a and the curve $x^2 = 4ay$.

    b) Prove that $\beta(m,n) = \frac{\overline{|(m)}\overline{|(n)}}{\overline{|(m+n)}}$.

20. a) Solve: $q = px + p^2$.

    b) Solve: $x(y-z)p + y(z-x)q = z(x-y)$.

21. a) Verify Stoke's theorem for $\vec{F} = \left(x^2 - y^2\right)\hat{i} + 2xy\,\hat{j}$ in the rectangular region in the xy plane bounded by the lines x=0, x=a, y=0, y=b.

    b) State and prove 'Weistrass inequality'.

22. a) Using Laplace transform, solve $\frac{d^2y}{dt^2} + \frac{dy}{dt} = t^2 + 2t$ given that y=4 and $y' = -2$ when t=0.

    b) State and prove Wilson's theorem.

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